Name ____________________ Date ____________

Day 5 – Arc Length and Area of a Sector

Today we will be exploring arc length and the area of a circle. Before we can introduce arc length we need to review.

Review: The arc length of the entire circle, or the distance around the circle is called what? circumference.

The formula for circumference is:

Circumference

$C = 2\pi r$ or $C = \pi d$

Where "r" represents the radius of the circle and "d" represents the diameter.

Practice reviewing how to calculate the radius or diameter of a circle below. Find the radius or diameter.

A. r = 6 ft
$d = 12 ft$

B. d = 15 in
$r = 7.5 in$

C. r = 16 cm
$d = 32 cm$

D. d = 40 m
$r = 20 m$

Practice reviewing how to calculate the radius or diameter of a circle when given the circumference. Find the radius and diameter.

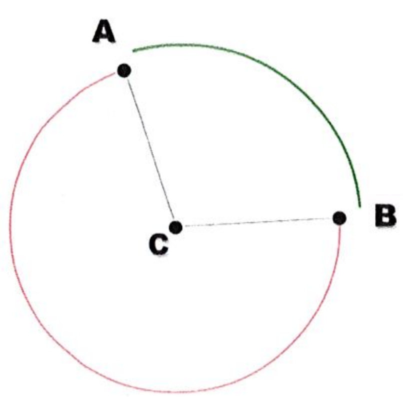
A. $C = 50\pi$ m
$d = 50 m$ $r = 25 m$

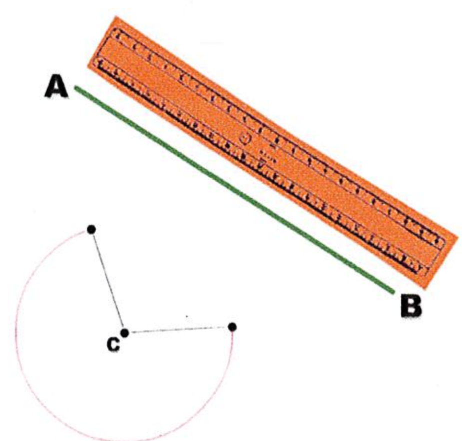
B. $C = 18\pi$ cm
$d = 18 cm$ $r = 9 cm$

C. $C = 8\pi$ ft
$d = 8 ft$ $r = 4 ft$

D. $C = 96\pi$ m
$d = 96 m$ $r = 48 m$

Now that we have reviewed circumference, we can introduce arc length. **Arc length** is a fraction of the circle's circumference and is measured in linear units.





The formula for Arc Length is as follows:

Arc Length

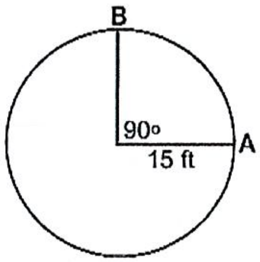
$$\text{arc length} = \frac{2\pi r\theta}{360}$$

Where "r" represents the radius and "θ" is the measure of the central angle.

When calculating arc length, you may be required to provide an exact answer or an approximate answer. An *exact answer* needs to be left in terms of "pi". An *approximate answer* can be in decimal format (where pi has been multiplied through).

Practice

Example: Find the length of arc BA.

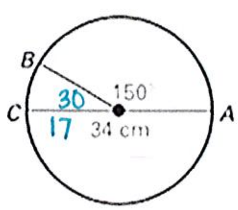


$\overset{\frown}{BA} = \frac{2\pi(15)(90)}{360}$

$\overset{\frown}{BA} = \frac{2700\pi}{360}$

$\boxed{\overset{\frown}{BA} = \underbrace{\frac{15}{2}\pi \text{ ft}}_{\text{exact}} \text{ or } \underbrace{23.56 \text{ ft}}_{\text{approx.}}}$

Example: Find the length of arc BC.



$180 - 150 = 30$

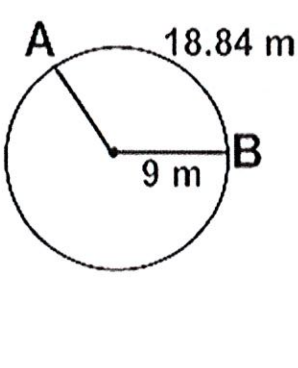
$34/2 = 17$

$\overset{\frown}{BC} = \frac{2\pi(17)(30)}{360}$

$\overset{\frown}{BC} = \frac{1020\pi}{360}$

$\boxed{\overset{\frown}{BC} = \underbrace{\frac{17}{6}\pi \text{ cm}}_{\text{exact}} \text{ or } \underbrace{8.90 \text{ cm}}_{\text{approx.}}}$

Example: Find the measure of arc AB. * looking for θ



$360 \cdot 18.84 = \frac{2\pi(9)\theta}{\cancel{360}} \cdot \cancel{360}$

$6782.4 = 2\pi(9)\theta$

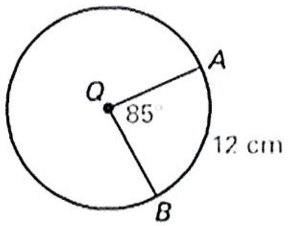
$\frac{6782.4}{56.52} = \frac{\cancel{56.52}\theta}{\cancel{56.52}}$

$120 = \theta$

$\theta = 120°$

$\boxed{m\overset{\frown}{AB} = 120°}$

Example: Find the circumference of Circle Q. $C = 2\pi r$



$360 \cdot 12 = \dfrac{2\pi r(85)}{\cancel{360}} \cdot \cancel{360}$

$\dfrac{4320}{85} = \dfrac{2\pi r(\cancel{85})}{\cancel{85}}$

$50.82 \approx 2\pi r$

$\boxed{C \approx 50.82\text{ cm}}$

Next, we will be exploring the area of a sector. In order to do that, we must recall the area of a circle. The area of a circle is the number of square units inside the circle.

Area
$A = \pi r^2$

Where "r" represents the radius of the circle.

Practice reviewing how to calculate the area or radius/diameter of a circle below. Leave your answers in terms of pi. Find the area, radius, or diameter.

A. r = 6 ft

$\boxed{d = 12\text{ ft}}$

$A = 6^2\pi$

$\boxed{A = 36\pi\text{ ft}^2}$

B. d = 18 in

$\boxed{r = 9\text{ in}}$

$A = 9^2\pi$

$\boxed{A = 81\pi\text{ in}^2}$

C. A = 121π in^2

$\sqrt{121} = 11$

$\boxed{r = 11\text{ in}}$

$\boxed{d = 22\text{ in}}$

D. A = 16π m^2

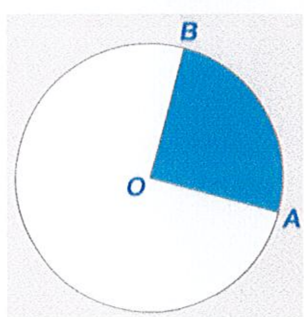
$\sqrt{16} = 4$

$\boxed{r = 4\text{ m}}$

$\boxed{d = 8\text{ m}}$

Now that we have reviewed the area of a circle, we can introduce the Area of a sector.

A **sector** of a circle is a region bounded by two radii and their intercepted arc.



The shaded region bound by $\overline{BO}$, $\overline{AO}$, and $\overset{\frown}{AB}$ is a sector.

The formula for the Area of a Sector is as follows:

Area of a Sector

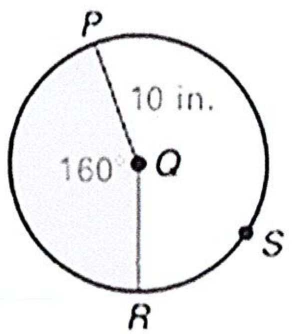
$$\text{area of a sector} = \frac{\pi r^2 \theta}{360}$$

Where "r" represents the radius and "θ" is the measure of the central angle.

When calculating the area of a sector, you may be required to provide an exact answer or an approximate answer. An *exact answer* needs to be left in terms of "pi". An *approximate answer* can be in decimal format (where pi has been multiplied through).

Practice

Example: Find the area of the shaded sector formed by angle PQR.



$$AOS = \frac{\pi(10^2)(160)}{360}$$

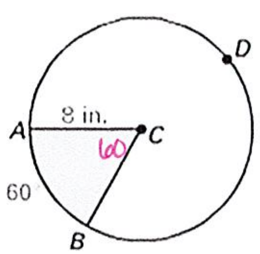
$$AOS = \frac{16000\pi}{360}$$

the area of the sector is

$\frac{400}{9}\pi \text{ in}^2$ or 139.63 in^2

exact approx.

Example: Find the area of the shaded sector formed by the angle ACB.



$$AOS = \frac{\pi(8^2)(60)}{360}$$

$$AOS = \frac{3840\pi}{360}$$

the area of the sector is

$\frac{32}{3}\pi \text{ in}^2$ or 33.51 in^2

exact approx.